Velocity / Rate of change solutions

1. $s(t) = -2t^2 + 7t$

① $\frac{s(t+h) - s(t)}{h} = \frac{-2(t+h)^2 + 7(t+h) - (-2t^2 + 7t)}{h}$

$= \frac{-2(t^2 + 2th + h^2) + 7t + 7h + 2t^2 - 7t}{h}$

$= \frac{\cancel{-2t^2} - 4th - 2h^2 + \cancel{7t} + 7h + \cancel{2t^2} - \cancel{7t}}{h}$

$= \frac{\cancel{h}(-4t - 2h + 7)}{\cancel{h}}$

② $\lim_{h \to 0} (-4t - 2h + 7)$

$= \boxed{-4t + 7 = V(t)}$

2. $s(t) = \frac{7}{t}$

① $\frac{s(t+h) - s(t)}{h} = \frac{\frac{7}{t+h} - \frac{7}{t}}{h} = \frac{\frac{7t}{t(t+h)} - \frac{7(t+h)}{t(t+h)}}{h}$

$= \frac{\frac{\cancel{7t} - \cancel{7t} - 7h}{t(t+h)}}{h} = \frac{\frac{-7\cancel{h}}{t(t+h)}}{\cancel{h}} = \frac{-7}{t(t+h)}$

② $\lim_{h \to 0} \frac{-7}{t(t+h)} = \frac{-7}{t(t+0)} = \boxed{\frac{-7}{t^2} = V(t)}$

3. $s(t) = \frac{2t-8}{t}$

① $\frac{s(t+h)-s(t)}{h} = \frac{\frac{2(t+h)-8}{t+h} - \frac{2t-8}{t}}{h}$

$= \frac{\frac{2t+2h-8}{t+h} - \frac{2t-8}{t}}{h} = \frac{\frac{2t+2h-8}{(t+h)}\cdot\frac{t}{t} - \frac{2t-8}{t}\cdot\frac{t+h}{t+h}}{h}$

$= \frac{\frac{2t^2+2th-8t-(2t^2+2th-8t-8h)}{t(t+h)}}{h}$

$= \frac{\frac{\cancel{2t^2}+\cancel{2th}-\cancel{8t}-\cancel{2t^2}-\cancel{2th}+\cancel{8t}+8h}{t(t+h)}}{h}$

$= \frac{\frac{8\cancel{h}}{t(t+h)}}{\cancel{h}} = \frac{8}{t(t+h)}$ ② $\lim_{h\to 0} \frac{8}{t(t+h)} = \boxed{\frac{8}{t^2} = v(t)}$

4. $s(t) = \sqrt{t+3}$

① $\frac{s(t+h)-s(t)}{h} = \frac{\sqrt{t+h+3}-\sqrt{t+3}}{h} \cdot \frac{\sqrt{t+h+3}+\sqrt{t+3}}{\sqrt{t+h+3}+\sqrt{t+3}}$

$= \frac{t+h+3-(t+3)}{h(\sqrt{t+h+3}+\sqrt{t+3})} = \frac{\cancel{t}+h+\cancel{3}-\cancel{t}-\cancel{3}}{h(\sqrt{t+h+3}+\sqrt{t+3})}$

$= \frac{\cancel{h}}{\cancel{h}(\sqrt{t+h+3}+\sqrt{t+3})} = \frac{1}{\sqrt{t+h+3}+\sqrt{t+3}}$

② $\lim_{h\to 0} \frac{1}{\sqrt{t+h+3}+\sqrt{t+3}} = \frac{1}{\sqrt{t+3}+\sqrt{t+3}} = \boxed{\frac{1}{2\sqrt{t+3}} = v(t)}$

5.

$$V(t) = 100{,}000\left(1 - \frac{t}{60}\right)^2$$

① $$\frac{V(t+h) - V(t)}{h} = \frac{100{,}000\left(1 - \frac{t+h}{60}\right)^2 - 100{,}000\left(1 - \frac{t}{60}\right)^2}{h}$$

$$= \frac{100{,}000\left[\left(1 - \frac{t+h}{60}\right)\left(1 - \frac{t+h}{60}\right) - \left(1 - \frac{t}{60}\right)\left(1 - \frac{t}{60}\right)\right]}{h}$$

$$= \frac{100{,}000\left[1 - \frac{2(t+h)}{60} + \frac{(t+h)^2}{60^2} - \left(1 - \frac{2t}{60} + \frac{t^2}{60^2}\right)\right]}{h}$$

$$= \frac{100{,}000\left[1 - \frac{t}{30} - \frac{h}{30} + \frac{t^2 + 2th + h^2}{3{,}600} - 1 + \frac{t}{30} - \frac{t^2}{3{,}600}\right]}{h}$$

$$= \frac{100{,}000\left[\cancel{1} - \cancel{\frac{t}{30}} - \frac{h}{30} + \cancel{\frac{t^2}{3{,}600}} + \frac{th}{1{,}800} + \frac{h^2}{3{,}600} \cancel{-1} + \cancel{\frac{t}{30}} - \cancel{\frac{t^2}{3{,}600}}\right]}{h}$$

$$= \frac{100{,}000\left[-\frac{h}{30} + \frac{th}{1{,}800} + \frac{h^2}{3{,}600}\right]}{h} = \frac{100{,}000\cancel{h}\left(-\frac{1}{30} + \frac{t}{1{,}800} + \frac{h}{3{,}600}\right)}{\cancel{h}}$$

② $$\lim_{h \to 0} 100{,}000\left(-\frac{1}{30} + \frac{t}{1{,}800} + \frac{h}{3{,}600}\right) = \boxed{100{,}000\left(\frac{-1}{30} + \frac{t}{1{,}800}\right) = \text{rate of change of } V}$$